EXAMINATIONS IN ELEMENTARY GEOMETRY.

IN the spring of 1870 a letter appeared in the columns of NATURE suggesting the formation of an Association which should have for its object the improvement of geometrical teaching. As to the great desirability of such improvement there could be no doubt. It was felt that one of the greatest obstacles in the way of effecting it was the nature of the examinations, and that no change in that nature was likely to be brought about except by the combined action of those who desired it. The idea took root, and a circular was drawn up in which the objects of the proposed Association were stated to be—

(1) To collect and distribute information as to the prevailing methods of instruction in geometry practised in this and other countries, and to ascertain whether the desire for change was general.

(2) To use its influence to induce examining bodies to frame their questions in geometry without reference to any particular text-book.

(3) To stamp with its approval some text-book already published, or to bring out a new one under its own auspices.

The movement led to a conference at University College in 1871, which resulted in the formation of the Association for the Improvement of Geometrical Teaching. Many difficulties beset the way of the desired improvement. In addition to all the arguments which may be used against radical change in any department of affairs, the uncomprising supporters of Euclid (or rather of Simson's edition of his works) had the very cogent one that a Commission appointed by the Italian Government to inquire into the state of geometrical teaching in Italy found it "so unsatisfactory, and the number of bad text-books so great and so much on the increase, as to compel them to recommend the adoption of Euclid pure and simple,"[1] and were able to quote the authority of distinguished French mathematicians as to the great merits of Euclid as compared with modern French writers. We need not wonder, then, and need perhaps scarcely regret, that the outward and direct progress of the Association towards the realization of the aims set forth in its programme was slow.

One of its first steps was to attempt to draw up a Syllabus of Geometry in place of Euclid's scheme of propositions, which might form the basis of future text-books. Members were requested to send in, and the Committee of Management to report upon, programmes of the subjects which a text-book ought to include. Eleven programmes were sent in, which agreed upon many fundamental points; and after much work by a sub-committee, and much discussion from time to time at the general meetings of the Association, a Syllabus of Geometry was at last published as the work of the Association. It is worthy of remark that a Committee of the British Association, containing such eminent mathematicians as Profs. Cayley, Clifford, H. J. S. Smith, and Sylvester, appointed for the purpose of considering the possibility of improving the methods of instruction in elementary geometry, and reappointed to consider this Syllabus, reported that "it appears to have been drawn up with such care and with such regard to the essential conditions of the problem as to render it highly desirable that it should be considered in detail by authorized representatives of the Universities and other great examining bodies of the United Kingdom, with a view to its adoption, subject to any modification which such detailed consideration may show to be necessary, as the standard for examinations."

The secretaries then applied to many of the principal examining bodies, submitting the Syllabus for their consideration, with a view to its adoption as the basis of examination.

The direct result of this application was small. It was felt that the non-existence of a text-book based on the Syllabus was a bar to its adoption, and a sub-committee was appointed to prepare proofs of the propositions. A text-book embodying these, and entitled "The Elements of Plane Geometry," was published by Messrs. Sonnenschein and Co. The appearance of Part II. of this work in 1886 seemed to afford a fitting opportunity for memorializing the Universities as to the advisability of relaxing their regulations. The following petition was drawn up, and received about 180 signatures, among which may be noticed those of Sir R. S. Ball, Prof. Chrystal, Prof. Henrici, Dr. Hirst, and Prof. Tait:—

"We, the undersigned members of the Association for the Improvement of Geometrical Teaching, and others interested in the teaching of elementary geometry, believing that greater freedom in the teaching of that subject than is consistent with a rigid adherence to the letter of Euclid's 'Elements' is highly desirable, would welcome such a change in the examinations in elementary geometry conducted by the Universities and other examining bodies as would admit of the subject being studied from text-books other than editions of Euclid, without the student being thereby placed at a disadvantage in those examinations."

The Council forwarded the petition to the Universities, praying them to take the subject of the petition into their favourable consideration, with a view to adapting the regulations for the examinations in elementary geometry conducted by the Universities so as to meet the desire for greater freedom felt by a large number of teachers, and supported by the judgment of many eminent mathematicians.

The Hebdomadal Council of the University of Oxford having referred the application of the Association to the Board of the Faculty of Natural Science, the Board reported as follows:—

(1) "That a rigid adherence to the ordinary text-books of Euclid should no longer be insisted on, but that a greater freedom of demonstration should be allowed, both in geometrical teaching and in examination.

(2) "That, nevertheless, Euclid's *method* should be required in all pass examinations in geometry in so far as that no axioms other than those of Euclid shall be admitted, and that no proof of a proposition be allowed which assumes the truth of any proposition which does not precede it according to Euclid's order.

(3) "That the University should not prescribe the use of any particular text-book or text-books."

This Report was adopted by the Hebdomadal Council on June 20, 1887.

The Senate of the University of Cambridge having referred to the Special Board for Mathematics, the Board reported as follows:—

"The majority of the Board are of opinion that the rigid adherence to Euclid's text is prejudicial to the interests of education, and that greater freedom in the method of teaching geometry is desirable. As it appears that this greater freedom cannot be attained while a knowledge of Euclid's text is insisted upon in the examinations of the University, they consider that such alterations should be made in the regulations of the examinations as to admit other proofs besides those of Euclid, while following, however, his general sequence of propositions, so that no proof of any proposition occurring in Euclid should be accepted in which a subsequent proposition in Euclid's order is assumed.

"To give effect to this view, the Board recommend that for Regulation 10 (2) of the previous examination (*Ordinationes*, p. 8) the following be substituted:—

"Elementary geometry, viz. the substance of the first three books, the definitions 1–10 of Book V., and the substance of the first nineteen propositions of the Sixth Book of Euclid's 'Elements.' Euclid's definitions will be

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[1] See First Annual Report of the A.I.G.T.

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required, and no axioms or postulates except Euclid's may be assumed. The actual proofs of propositions as given in Euclid will not be required, but no proof of any proposition occurring in Euclid will be admitted in which use is made of any proposition which in Euclid's order occurs subsequently."

The subject came on for discussion in the School on January 26, 1888, and a grace finally passed the Senate (March 8, 1888), making the alteration in the regulations for the previous examination recommended in the Report.

The Council of the Association then forwarded an Address to Her Majesty's Civil Service Commissioners, drawing their attention to the recent Reports of the University Boards, and asking for corresponding alterations in the regulations for the Woolwich and Sandhurst examinations, as follows:

"In view of the above facts, the Council feel themselves justified in inviting the attention of Her Majesty's Civil Service Commissioners to the desirability of a change in one of the existing regulations for the examinations conducted under their authority, especially those for admission to Sandhurst and Woolwich. By this a candidate is required to 'satisfy the Commissioners in Euclid, Books I.-IV. and VI.' The Council, having reason to believe that this regulation is very generally understood to imply that Euclid's text is required without any but mere verbal variations, beg to suggest that the subject should be defined as 'Elementary Geometry to the extent of Books I.-IV. and VI. of Euclid's "Elements,"' with a note to the effect that 'any proofs will be admitted which are themselves sound, do not assume other axioms than those of Euclid, and are not inconsistent with the logical sequence of Euclid's theorems.'

"The Council would prefer to have no particular text-book prescribed, and to have the candidate's general knowledge of geometry tested, rather than his power of reproducing Euclid's text.

"While making this request, however, the Council wishes to express its acknowledgment of the high character of the papers in Euclid set by the Civil Service Examiners, and particularly of the frequent introduction of stimulative exercises on the book work set.

"The Council would esteem it a favour if they were allowed to express their views on this important question more at large by a deputation from their body at an interview with the Commissioners."

The following reply was received from the Commissioners:—

"*Civil Service Commission, June* 30, 1888.

"Sir,—I am directed by the Civil Service Commissioners to acknowledge the receipt of a circular signed by you in behalf of the Council of the Association for the Improvement of Geometrical Teaching, requesting that certain changes may be introduced into the regulations for the examinations held under their authority, and in particular those for admission to Sandhurst and Woolwich;

"And, in reply, I am to acquaint you, for the information of the Council, that the regulations for the military examinations are framed and issued by the military authorities, and that the Commissioners have no power to alter them, but that the interpretation which the Commissioners have given them has been substantially in accordance with the views expressed by the Cambridge Special Board for Mathematics and the Oxford Board of the Faculty of Natural Science. They have, that is to say, instructed their occasional examiners to make no deduction from the marks allotted to a question because a candidate substitutes another proof for Euclid's, if this proof is a sound one and keeps to Euclid's sequence of propositions. To remove any misconception which may exist, the Commissioners propose to introduce a note to this effect in the examination papers. It will be understood, however, that they do not intend thereby to fetter their discretion of changing their practice whenever they may consider it expedient.

"In conclusion, while thanking the Council for their offer to express their views on this question more at large by the medium of a deputation from their body, the Commissioners think that the agreement of their opinion on this matter renders it unnecessary that the members of the Council should give themselves the trouble of an interview.

"I have the honour to be, Sir,
"Your obedient servant,
"E. POSTE."

Doubtless these concessions are in form the sorriest minimum of change that could well be granted; but the spirit of the Reports is excellent, particularly if examiners allow themselves, as it is quite open for them to do, to interpret "Euclid's order" as his *logical* and not merely his *numerical* order. Probably, if there is a strong desire for further relaxation generally felt and expressed by teachers, there will be no great difficulty in getting it granted; but it is by no means clear that such a desire exists at present to a sufficient degree to bring pressure to bear on the authorities. One does not easily see why the sequence of the Syllabus and the "Elements" should not have been granted as an alternative to that of Euclid, as is done by the University of Edinburgh. These works are the outcome of considerable discussion and deliberation by practised teachers, and it seems scarcely right to ignore them so entirely. As far as Cambridge is concerned, there is one detail of reform which might be effected without going beyond the terms of the grace of the Senate; *i.e.* the authorization of a list of additional propositions not contained in Euclid's text, to be freely used in the demonstration of all propositions subsequent to the position they may be regarded as occupying in Euclid's sequence. Such a list need not be long, and its adoption would be a boon to those teachers who wish to use the Syllabus.

ELECTROSTATIC MEASUREMENT.[1]

THE number of electrostatic units of potential or electromotive force in the electro-magnetic unit of potential is essentially a velocity, and experiments have proved it to be so nearly equal to the velocity of light that from all the direct observations hitherto made we cannot tell whether it is a little greater than, or a little less than, or absolutely equal to, the velocity of light.

Sir W. Thomson is engaged now in making a set of electrometers which will measure by electrostatic force potentials of from 40 volts to 50,000 volts. The standardization of these instruments up to 200 or 300 volts is made exceedingly easy, by aid of his centiampere balance and continuous rheostat, with a voltaic battery of any kind, primary or secondary, capable of giving a fairly steady current of $\frac{1}{10}$ of an ampere through it and the platinoid resistance in series with it. The accuracy of the electro-magnetic standardization, within the range of the direct application of this method, is quite within $\frac{1}{20}$ per cent. A method of multiplication by aid of condensers, which was explained, gives an accuracy quite within $^1/_5$ per cent. for the measurement in volts up to 2000 or 3000 volts; and with not much less accuracy, by aid of an intermediate electrometer, up to 10,000 volts. Such a potential as 10,000 volts is convenient for measurement by an absolute electrostatic balance, which was fully explained in the lecture, and illustrated by a drawing. But hitherto he has not been able to make sure of the absolute accuracy of the electrostatic balance to closer than $^1/_2$ per cent. The results of a great number of measurements made in the Physical Laboratory of the University of Glasgow during the last

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[1] Abstract of part of Sir William Thomson's Royal Institution Lecture of Friday evening, February 8, relating to the velocity of light.

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